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Theory and experiments of concentration polarization and ion focusing at microchannel and nanochannel interfaces†

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In this tutorial review aimed at researchers using nanofluidic devices, we summarize the current state of theoretical and experimental approaches to describing concentration polarization (CP) in hybrid microfluidic–nanofluidic systems. We also analyze experimental results for these systems and place them in the context of recent theoretical developments. We then extend the theory to explain the behavior of both positively and negatively charged, low-concentration, analyte species in systems with CP. We conclude by discussing several applications of CP in microfluidics.

1. Introduction

A variety of fabrication techniques are now available to construct nanofluidic devices and interface them with microfluidic systems.1–3,8 This has motivated the study of nanofluidic devices as modules within microfluidic systems to, for example, preconcentrate analytes1,5,6 or detect and analyze DNA.7 However, application of an electric field across a nanofluidic device creates regions of enriched and depleted (net neutral) ion concentration, an effect called concentration polarization (CP). CP can change the conductivity and electric field in adjoining microchannels by orders of magnitude8 which, in turn, influences sample transport throughout these systems.9,10 Recent work has shown that CP enrichment and depletion zones can propagate through hybrid microchannel–nanochannel devices and profoundly affect the global behavior of the system.2,11

A large number of experimental studies1–3,5,8,10,12–27 have addressed the influence of CP on the transport of an analyte ion with a concentration relatively low compared to the concentration of background ions in hybrid microfluidic–nanofluidic systems. To date we know of only two papers which have attempted to explain and categorize several of these experimental results. However, the first is only a preliminary work we published on this topic,10 and the second consisted of a (transient) one-dimensional computational model.9 In this paper we extend recent theoretical developments11 to provide a more generalized analytical framework for understanding the effects of CP on both background and analyte ion transport in hybrid microfluidic–nanofluidic systems.

We first review nanochannel physics as it relates to CP (section 2). Second, we briefly discuss historical CP work and introduce the recent literature (section 3). Third, we summarize the analytical CP theory recently presented by Mani et al.11 (section 4). Sections 5 and 6 then describe the effects of CP on cationic and anionic analytes, respectively, in systems with...
negatively charged nanochannel walls (we will focus all of our discussions on the case of negatively charged walls). Finally, section 7 briefly discusses several applications of CP effects in microfluidics.

Note that in this review, we consider electrokinetic CP in microchannel systems caused by forcing a current through a nanochannel or a nanoporous membrane. There are at least two other physical phenomena which are commonly referred to as concentration polarization and which are not the topic of this review. These include solute buildup at a (steric) filter membrane or performance losses in a fuel cell due to the concentration of reactants.

In our discussion of low concentration analyte ions, we will examine electrokinetic preconcentration at microchannel–nanochannel interfaces. However, we note that other preconcentration modes are possible. For example, Foote et al. fabricated silica nanoporous membranes and preconcentrated proteins using size exclusion. Similarly, Meagher et al. used a size-exclusion preconcentrator to improve the limit-of-detection of an on-chip immunoassay from 300 pM to <10 pM. For a more thorough overview of size exclusion techniques using nanoporous membranes and nanochannels, see the recent review by Han et al. We note, however, that CP has been observed to influence the performance of nanoporous membranes (including size-exclusion-based systems) under conditions in which the nanoporous membrane is charged. Hlushkou et al. reported on the preconcentration efficiencies at charged and uncharged nanoporous membranes and found that much higher field strengths are required to achieve the same preconcentration factor with charged membranes due to the CP depletion region. This depletion zone acts as a large electrical resistance, increasing the required applied electric potential. In a paper on an integrated size-exclusion preconcentrator, Hatch et al. reported a decrease in current with time due to growth of an anode-side, CP depletion zone; this had an adverse effect on repeatability of the subsequent separation in polyacrylamide sieving matrix. A similar effect has been observed in DNA capillary separation though a gel.

CP effects in membrane preconcentration can perhaps be mitigated by avoiding trans-membrane current during a separation step (following a size-exclusion preconcentration) or by applying current through the membrane only for a short time.

2. Nanochannel physics and CP

The physics of electrokinetics in nanochannels are discussed elsewhere and will not be reviewed at length here. Instead, we present here brief scaling arguments which highlight key physical effects. CP occurs when current flows into or out of an electric double layer (EDL) shielding a charged surface. The common case is when ionic current passes from a microchannel into a nanochannel. In the nanochannel, a significant portion of the current is carried by the EDL which, unlike the bulk solution, has an appreciably different number of negative versus positive charges. This deviation from bulk conductance is well known and has been characterized experimentally. We can use a formulation of this deviation from bulk conductance to show that the primary parameter that governs CP is a type of Dukhin number, relating bulk and surface conductances. This is consistent with the analytical model we presented in ref. 11 in which a type of Dukhin number appears as a nondimensional parameter in the differential equations governing CP. This is also consistent with our experimental results showing that a Dukhin number can be used to accurately predict the behavior of CP enrichment and depletion regions.

Neglecting for the moment current due to diffusive fluxes, ion current through a wide, shallow (micro or nano) channel can be written as a sum of advection and electromigration current as:

\[
I = w \sum_i \left[ \int_0^h (Fz_i c_i(y) U_{\text{bulk}}(y)) dy + \int_0^h (F z_i^2 c_i(y) E) dy \right]
\]

where \(w\) is the width of the channel; \(h\) is the channel height; \(F\) is Faraday’s constant; \(z_i\) and \(\nu_i\) are the valence and mobility of the \(i\)th species; \(c_i\) is the concentration of the \(i\)th species which can vary with channel location; \(E\) is the external electric field; and \(U_{\text{bulk}}\) is the bulk velocity. In the thin electric double layer (EDL) limit the total channel conductance, \(G_{\text{tot}} = I/E\) can be estimated for a symmetric electrolyte as:

\[
G_{\text{thinEDL}} \approx 2F^2 z_i^2 c_0 w h
\]

where \(c_0\) is the concentration outside of the EDL. Eqn (2) is the definition of bulk conductance, so \(G_{\text{thinEDL}} = G_{\text{bulk}}\). Similar to the analysis of Stein et al. and for simplicity, we impose a boundary condition of constant and uniform wall charge, \(\sigma\), and take the limit of eqn (1) for thick EDLs (i.e., the limit of \(h/\lambda_D \ll 1\)), we obtain an expression for the surface conductance:

\[
G_{\sigma} \approx |\sigma| F z_i w \left( 1 + \frac{|\sigma|h}{6F z_i \eta} \right)
\]
In eqn (3), the second term in parentheses indicates the effects of electroosmotic transport on current and is briefly derived in Appendix I. Schoch et al.\textsuperscript{36} modeled the total conductance of a nanochannel as the sum of a bulk conductance term (as in eqn (2)) and a surface conductance term (as in eqn (3)) but without the advective current component and found a good fit to experimental data. Following this approach, and neglecting the advective current contribution, we can take the ratio of eqn (2) to the leading term of eqn (3) to obtain a good indication of the relative importance of bulk conductance, $G_{\text{bulk}}$, to surface conductance, $G_{\sigma}$:

$$\frac{G_{\text{bulk}}}{G_{\sigma}} \approx \frac{F \sqrt{\zeta \sigma}}{d}. \tag{4}$$

Eqn (4) is an inverse Dukhin number for a symmetric electrolyte, and is equal to the more general parameter used by Mani et al.\textsuperscript{11} and has also been noted by Kim\textsuperscript{1} in descriptions of CP. This scaling is in fact not nearly as useful as $F \sqrt{\zeta \sigma}/d$ values of 10 or more. For example, strong CP effects have been reported for $h/\lambda_d$ values of greater than 100\textsuperscript{26}.

3. Concentration polarization models and experiments

CP has been studied extensively in the context of colloids\textsuperscript{37,39} and membranes.\textsuperscript{30,42} In early work on membrane CP, Block and Kitchener\textsuperscript{40} studied nanoporous materials and speculated on the role of water dissociation in the low-concentration depletion regions. In a seminal CP contribution, Dukhin and Shilov\textsuperscript{39} presented a simplified description of CP based on a model with very thin EDLs that carry current due to surface conduction. Their model showed that CP occurs when electric field lines cross from net neutral, bulk solution, into electrical double layers. More recent work has shown that electrokinetic instabilities can cause systems with nanoporous membranes to have higher ("overlimiting") conductance than the limit dictated by ion diffusion.\textsuperscript{43} Tallarek and coworkers have examined CP in beds of porous beads and glass monoliths.\textsuperscript{13,27} See the review by Hölzl and Tallarek\textsuperscript{44} for more information on CP around nanoporous membranes, packed beds, and glass monoliths.

In microfluidic systems, Pu et al.\textsuperscript{14} showed the first reported visualization of CP near nanochannels and presented a qualitative model describing CP. Major themes of research on microfluidic–nanofluidic interfaces since this work have been the study of recirculation zones and the effects of CP on the concentration of ionic species. We will briefly introduce this research in sections 3.1 and 3.2, before going into much greater detail on CP effects on ion concentration in sections 4, 5 and 6.

3.1 Recirculation effects

Recirculation near microfluidic–nanofluidic interfaces has been noted in several studies\textsuperscript{8,11,19,45,46} and at least three different physical origins of this recirculation have been noted. Park et al.\textsuperscript{45} presented an analytical model suggesting two different mechanisms for the formation of vortices at a gradual microchannel to nanochannel constriction. Though this model neglects the effects of CP on the background electrolyte concentration, it does capture two possible mechanisms for vortex generation at a microchannel–nanochannel interface. First, surface conductance causes the ratio of ionic current to bulk flow to vary with changes in the channel cross-sectional area. Second, the effects of finite EDLs cause the electroosmotic flow (EOF) velocity profile to be non-uniform across a microchannel–nanochannel interface. Both of these effects create internal pressure gradients. The analysis of Park et al.
demonstrates that these internal pressure gradients can create vortices at the channel centerline, along the channel walls, or in both regions.\textsuperscript{45} Similar vortices were noted in the computational results of Mani \textit{et al.}\textsuperscript{11} and Postler \textit{et al.}\textsuperscript{46} The latter two studies found that an adverse pressure gradient can lead to local flow reversal and the formation of eddies near microchannel–nanochannel interfaces. Postler \textit{et al.}\textsuperscript{46} also examined the coupling of CP and electroosmotic flow.\textsuperscript{45}

The work of Zaltzman, Rubinstein, and co-workers\textsuperscript{41,43} suggests that electrokinetic flow instability is another possible mechanism for the formation of vortices at a microfluidic–nanofluidic interface. This work has focused on nanoporous membrane systems in which current is observed to initially increase linearly with increasing applied electric field up to a point where current through the membrane becomes limited by diffusion through the depletion region. This is called the “limiting current” regime. Zaltzman, Rubinstein, \textit{et al.}\textsuperscript{41,43} demonstrated that a fluid instability is responsible for mixing the depletion region (with fluid regions of less depleted ion density) at electric fields beyond the limiting current regime. Mixing due to instability at high electric fields reduces the resistance of the depletion region, resulting in a notably higher current through the nanoporous membrane, which is called “overlimiting current.” See Zaltzman \textit{et al.}\textsuperscript{43} for more details on overlimiting current in membrane systems. Kim \textit{et al.}\textsuperscript{19} studied overlimiting currents in microchannel–nanochannel systems and found that the observed behavior of analyte molecules in the depletion region was correlated to changes from limiting to overlimiting current behavior. Recent work by the same group\textsuperscript{8} studied the physics of the depletion region in a nanofluidic preconcentrator where recirculation was observed. From measurements made using microfabricated electrodes the authors estimated the electric field in the depletion region to be up to 33 times higher than the nominal electric field applied to their device.

3.2 Work describing ion concentration

Plecis \textit{et al.} described different focusing and stacking\textsuperscript{3} regimes of analyte species using a computational model which we will discuss in more detail below.\textsuperscript{9} In simultaneous work, we presented stacking and separation of two low concentration anionic species on a CP enrichment shock.\textsuperscript{10} These two species were assumed to be present in low concentration relative to the background electrolyte (BGE) so that they do not affect the dynamics of CP and the axial electric field. Zangle \textit{et al.}\textsuperscript{10} included an analytical model for stacking of an anionic species

\textsuperscript{3} Consistent with Bharadwaj \textit{et al.},\textsuperscript{47} we use focusing to describe the condition where there is a focal point for the analyte. A focal point is a point or region in some reference frame where the drift velocity of the focused species changes sign so that sample is driven to the focal point. We use “stacking” to describe the less stringent situation where an analyte drift velocity decreases in magnitude (causing an increase in concentration) as the analyte migrates through some stacking region containing an electric field gradient.
on the enrichment shock. In the following sections, we will present a generalization of this model which describes ion behavior at each region indicated in Fig. 2. Fig. 2 shows computations for the concentration of a background electrolyte (dominant ion) in a microchannel–nanochannel system resulting from CP as black lines. Green peaks indicate schematically possible focusing or stacking locations of an anionic analyte.

We also recently presented an analytical description of the behavior of the background electrolyte (BGE) in microchannel–nanochannel systems. In particular, our model shows behavior of the background electrolyte (BGE) in microchannel–nanochannel systems. We tested a simplified model of charged species transport in a series microchannel–nanochannel–microchannel system. We supported our model with experiments and quantitative limits of the associated regimes. In order to provide a framework for comparing as many experimental cases as possible, we will use the most general analytical theory of CP in a channel system to date, which was presented by Mani et al.

We will first look at experimental results in the context of this binary electrolyte theory’s predictions of CP propagation. Then we will extend the theoretical results to predict the behavior of specific low concentration counter-ionic and co-ionic analyte species in a microchannel–nanochannel system.

4. Propagating vs. non-propagating CP and nanochannel ion concentration

In this section, we summarize the physical regimes which lead to the propagation of CP zones away from nanochannels and regimes which lead to non-propagating CP. In a two-paper series, Mani et al. and Zangle et al. developed and tested a simplified model of charged species transport in a series microchannel–nanochannel–microchannel system. We showed that concentration polarization (CP) enrichment and depletion zones will propagate as shock waves if the following condition is satisfied:

\[ c_{0,d}h_n^* \leq \max(\nu_2^*, 2\nu_2^* - 1) \tag{5} \]

where \( c_{0,d}h_n^* = (\nu_1z_1 - \nu_2z_2)Fh_n\delta_{0,d}/(-2\nu_1\sigma) \) is an inverse Dukhin number which describes the ratio of bulk to surface conductance, and \( \nu_2^* = \nu_2z_2F\eta/\xi_h \) is the mobility of the ion (to the wall charge) nondimensionalized by the electroosmotic mobility. Here, \( c_{0,r} \) is the BGE reservoir concentration. \( h_n \) is the nanochannel height; \( \nu_1 \) and \( \nu_2 \) are the mobilities of the positive and negative species; \( z_1 \) and \( z_2 \) are the valences of the positive and negative species, \( \sigma \) is wall charge, \( \xi \) is the zeta potential, \( F \) is Faraday’s constant, \( \varepsilon \) is permittivity and \( \eta \) is viscosity.

The enrichment and depletion shock waves predicted by Mani et al. are sharp, propagating boundaries between regions of the microchannel where concentration has been influenced by the nanochannel and regions which are still at the initial, reservoir concentration. These shocks are clearly seen in plots of concentration versus time and axial dimension as in Fig. 3 below. In Fig. 3, the depletion shock on the left separates the low concentration (black) depletion region from undisturbed regions. Similarly, the enrichment shock on the right is the moving boundary between the high concentration (white or yellow) region and the undisturbed region. For a constant applied current through the nanochannel, these shocks will propagate at constant rates to the reservoirs at the end of the microchannels.

Mani et al. gave predictions of both the dynamics and steady state values of ion concentrations in the microchannel and the nanochannels. For non-propagating CP, the final values achieved in the microchannel and nanochannel are

\[ c_{0,d} = c_{0,e} = c_{0,r} \tag{6} \]

where the subscripts \( d, e, r \) and \( n \) refer to the regions shown in Fig. 2. For propagating CP, there are two relations corresponding to two ranges of \( \nu_2^* \). For propagating CP with \( \nu_2^* < 1 \),

\[ c_{0,d} = c_{0,n} = 0 \text{ and } c_{0,e} = \nu_2^*/h_n^* \tag{7} \]

Fig. 3 Experimental spatiotemporal data showing enrichment and depletion shock waves emanating from both sides of a nanochannel between two microchannels. This plot shows width-averaged fluorescence intensity (using a colormap) as a function of axial position (x-axis) and time (y-axis). An electric field was applied from left (anode) to right (cathode). In this constant-current experiment, the enrichment region spreads towards the cathode (on the right) at constant velocity and the depletion region spreads at constant velocity towards the anode (to the left). The nanochannel is located between \( x = -100 \) and 0 µm. Adapted with permission from Zangle et al. Copyright 2009 American Chemical Society.
While for propagating CP with \( \nu_2^* \geq 1 \), we have

\[
\begin{align*}
\mathcal{c}_0^* & = \left( \frac{\nu_2^* - 1}{\mathcal{h}_n^*} \right)^2, \\
\mathcal{c}_{0,n}^* & = \left( \frac{\nu_2^* - 1}{\mathcal{h}_n^*} \right) \mathcal{c}_0^* \\
\mathcal{c}_{0,h}^* & = \left( \frac{\nu_2^* - 1}{\mathcal{h}_n^*} \right) \mathcal{c}_0^*.
\end{align*}
\]

(8)

The final concentrations inside microchannels and nanochannels given by eqn (6)-(8) in turn establish the electric field distribution in a microchannel–nanochannel systems with CP. In the following sections, we will use these results to predict the behavior of species which are in low-concentration relative to the background electrolyte (BGE) and which are either counter- or co-ions to the wall charge. In effect, these final ion concentration values will in part determine if and where a low concentration analyze species focuses or stacks.

The existence condition for propagating CP defined by eqn (5) is plotted as a solid line in Fig. 4. The region below this phase line indicates conditions which lead to CP with propagation, while the region above denotes non-propagating CP. The phase line is plotted here over the range of most propagation, while the region above denotes non-propagating CP. The parameter \( c_{0,h}^* \), a nondimensional concentration (inverse Dukhin number), is the major parameter determining propagating vs. non-propagating CP. The parameter \( c_{0,h}^* \) varies over many orders of magnitude, while \( \nu_2^* \) is typically of order unity. Therefore, to avoid the effects of propagating CP, a rule of thumb is to keep \( c_{0,h}^* \gg 1 \). If propagating CP is desirable for operation of a microchannel-nanochannel device, as is the case with some nanofluidic preconcentrators, then the rule of thumb is to insure that \( c_{0,h}^* \ll 1 \). In cases where \( c_{0,h}^* \sim 1 \) the designer should either consult the analytical theory or perform a computation using the specific conditions to be used.

As noted earlier, this theory assumes infinite PeL/h. Mani et al. provide a brief description of the effects of finite PeL/h on propagating CP. PeL/h effects may play a role in some reported observations including, for example, enrichment reports indicating non-propagating CP. Zeta potentials for SiO$_2$, PET and PDMS were estimated using the data and curve fits in Kirby et al. Zeta potential for HEMA hydrogel was estimated using data from Kuo et al. and You et al. and a curve fit of the form \( \zeta = A \cdot \log(c) + B \). Zeta potential of Nafion 117 was estimated using data from Ge et al. and Daiko et al.

Fig. 4 shows that the analytical theory of Mani et al. is a fairly good predictor of propagation behavior despite the simplifying assumptions made in deriving the model (for instance, the model assumes EDLs which are infinitely thin but which carry significant current, therefore, the definition of \( \nu_2 \) does not include any nanochannel height effects). Because the model is presented in terms of two non-dimensional parameters, it is useful as a design tool for a wide range of experimental conditions. Fig. 4 also shows that \( \mathcal{c}_0^*, \mathcal{c}_{0,n}^*, \mathcal{c}_{0,h}^* \), a nondimensional concentration (inverse Dukhin number), is the major parameter determining propagating vs. non-propagating CP. The parameter \( \mathcal{c}_0^*, \mathcal{c}_{0,n}^*, \mathcal{c}_{0,h}^* \) varies over many orders of magnitude, while \( \nu_2^* \) is typically of order unity. Therefore, to avoid the effects of propagating CP, a rule of thumb is to keep \( \mathcal{c}_0^*, \mathcal{c}_{0,n}^*, \mathcal{c}_{0,h}^* \gg 1 \). If propagating CP is desirable for operation of a microchannel-nanochannel device, as is the case with some nanofluidic preconcentrators, then the rule of thumb is to insure that \( \mathcal{c}_0^*, \mathcal{c}_{0,n}^*, \mathcal{c}_{0,h}^* \ll 1 \). In cases where \( \mathcal{c}_0^*, \mathcal{c}_{0,n}^*, \mathcal{c}_{0,h}^* \sim 1 \) the designer should either consult the analytical theory or perform a computation using the specific conditions to be used.

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\[ \text{Most reported experiments show visualization of a fluorescent species in low concentration relative to a non-fluorescent background electrolyte. Therefore, in such cases we looked for evidence of propagating or non-propagating CP of the background electrolyte by observing the behavior of the fluorescent analyze species. The expected behaviors for low concentration (analyze) species in propagating and non-propagating CP are described in sections 5 and 6 below.} \]
regions which first propagate and then stop in series microchannel–nanochannel–microchannel devices. Additionally, Plecis et al. showed that the applied potential is an important component of the regime ‘map’ of possible preconcentration modes. This suggests that, in addition to the two nondimensional parameters considered in this description of the background electrolyte, \( \nu_i^2 \) and \( c_i^0, h_i^e \), a third nondimensional parameter related to diffusion effects may be important. Based on the results of Plecis et al. and the description of Peclet number effects in Mani et al., we suggest that this additional parameter may be a Peclet number, which likely controls the thickness of enrichment and depletion shocks.

In the next sections we will extend the results from Mani et al. to describe the behavior of ionic species which are in low concentration relative to the background electrolyte (BGE). We will assume that the concentration of these ions is low enough (relative to the BGE) that they do not affect the electric field. We will call these species cationic or anionic analytes and, for simplicity, will only consider a system with negative wall charge. However, our approach is easily extendable to a system with positive wall charge.

5. Stacking of cationic analytes

In this section, we present a model to predict the behavior of a cationic analyte (a positively charged species which is in low concentration relative to the background electrolyte). Several investigators have reported experiments with low concentrations of positively charged Rhodamine 6G12,14,19,23,24,27 or Rhodamine 123 in a system with negatively charged nanochannel walls. Therefore, in this section we will analyze systems using these two fluorescent dyes as representative cationic analytes to test the analytical model predictions.

Cationic analytes electromigrate in the direction of the bulk flow and, so, in a linear microchannel–nanochannel system with negative wall charge, the cation’s total species velocity in the frame of reference of an interface (the sum of the electrophoretic and bulk velocities, \( U_{\text{pen}} = U_{\text{bulk}} \)) is always positive; and so the cation cannot focus in a microchannel–nanochannel system with CP. Cationic analytes, however, can stack (i.e., increase in concentration as they traverse some interface due to an increase in total species velocity, \( \nu_i^2 \)) or undergo electromigration dispersion (decrease concentration as they traverse some interface due to an increase in total species velocity, \( \nu_i^1 \)). We describe the behavior of cationic analytes by following an approach similar to that of Chambers et al. who analyzed the analogous situation of ions which move through but do not focus in isotachophoretic (ITP) zones. Our analysis uses the analytical results of Mani et al. to describe the background electrolyte (BGE) concentration and electric field everywhere in a microchannel–nanochannel–microchannel system. Then, we determine the concentration of cationic species which is required to balance the total flux across each interface in the system. Note that in this system the only interfaces where the total species velocity can change are the depletion shock, microchannel–nanochannel interfaces and the enrichment shock as shown in Fig. 5. Because cationic species move from the anode to the cathode, we assume that cations of interest start at the anode-side microchannel where the concentration is equal to the reservoir concentration, and proceed towards the cathode. For simplicity we assume also that the valence of the cation is the same as the valence of the cation in the BGE and that the mobility of each species in the system is constant (e.g., we do not consider ionic strength or \( \text{pH} \) effects on mobility). This analysis is presented in detail in Appendix II.

In Fig. 5 we provide the resulting relations from the analysis in Appendix II. The results describe the area-averaged stacking ratio of a cationic analyte species, \( i \), between each region of the microchannel-nanochannel system and the reservoir concentration. In typical fluorescence microscopy, the area averaged stacking ratio is directly related to the measured increase in fluorescence intensity inside the channel. Therefore, the stacking ratio is the experimentally accessible quantity. Stacking ratios in each region of the microchannel–nanochannel–microchannel system are given in terms of the key system parameters: channel height ratios, nondimensional mobilities, and Dukhin number, \( 1/c_i^0, h_i^e \). These are typically all known quantities.

The results shown in Fig. 5 indicate that a cationic analyte may be used to trace the development of the enrichment and depletion fronts. In the depletion region (first column of equations in Fig. 5), the local-to-reservoir cationic stacking ratio is inversely proportional to the nondimensional microchannel height, \( h_i^e \). \( h_i^e \) is typically very large, therefore we expect the concentration of a cationic analyte to approach zero inside the depletion region. This is similar to the behavior of the depletion region background electrolyte concentration which also approaches zero, making cationic analytes suitable for tracking the extent of the depletion region. In the enrichment region, the cationic analyte stacking ratio (rightmost column of Fig. 5), exactly matches the increase in the background electrolyte (eqn (7) and (8)). This makes a cationic tracer suitable as to both track the extent of and measure the concentration inside the enrichment region. Finally, the cationic stacking ratio inside the nanochannel is a function of both the analyte mobility, \( \nu_i^1 \), and the BGE properties. Therefore, the cationic stacking ratio is not a simple measurement of the BGE concentration inside the nanochannel.

Experimental results from Pu et al.14 showing the behavior of Rhodamine 6G in a Borofloat glass device with a 60 nm deep nanochannel are shown in Fig. 6(a)–(e), below. In these experiments an electric field was applied from right to left along a nanochannel (not visible) joining the two U-shaped microchannels shown in the figure (the anode and cathode are respectively on the right and left). As predicted by the results in Fig. 5, the concentration on the cathode side of the nanochannel (where we expect the enrichment region) increases and the concentration on the anode side decreases. Furthermore, the case where fluorescein is used as an analyte...
molecule (Fig. 6(f)–(j)) shows an enriched region which may be starting to separate from the nanochannel interface, in contrast to the Rhodamine 6G enrichment region which stays attached to the interface. This is consistent with the prediction that Rhodamine 6G, as a counter-ionic analyte, should trace the BGE enrichment region. Computational model results of background electrolyte behavior show that CP creates a net-neutral enrichment region which remains attached to the cathode-side interface.2,9,21 The observed behavior of fluorescein in these experiments is consistent with simulation results for an anionic analyte9,21 and will be discussed in more detail below.

As per Fig. 5, the concentration of a counter-ionic dye will be very low in the depletion region, making it suitable for tracking the edge of the depletion region over time in experiments. Consistent with this, Kim et al.19 used Rhodamine 6G combined with fluorescent beads to image CP-induced vortices at the boundary of the depletion region. Experiments by Huang and Yang24 using Rhodamine 6G as a counter-ionic tracer showed similar results to those in...
Fig. 6(a)–(e) and also used a cationic dye to trace the extent of the depletion region. Additionally, Huang and Yang performed numerical simulations of the flow in the depletion region using a constant zeta potential model. This model found that recirculation occurs due to a change in the degree of EDL overlap inside the depletion region, which lowers the overall bulk velocity. They found that the pattern of the edge of the depletion front imaged using Rhodamine 6G correlated overall bulk velocity. They found that recirculation occurs due to a change in the degree of EDL overlap inside the depletion region using a constant zeta potential model. This model formed numerical simulations of the flow in the depletion region. Additionally, Huang and Yang performed experiments with negatively charged walls. Here we extend the results of preliminary work we have published on this topic as well as the analysis of a recent paper by Plecis et al. Plecis et al. identifies four focusing regimes for anionic analytes and presents a computational model which captures phase boundaries based on surface charge, \( \sigma \), and applied potential. We here start in section 6.1 by deriving results for the behavior of anionic species using the analytical theory of Mani et al. Using an analytical model allows us to present a generalized regime map for arbitrary buffer and surface chemistry.

In sections 6.2–6.5, we compare both the analytical model based on Mani et al. and the computational results of Plecis et al. to published experimental results across a wide variety of conditions. We address separately each interface in a microchannel-nanochannel system with concentration polarization. Starting from the left side of Fig. 7 we will look at anionic analyte behavior on the depletion shock (section 6.2), the depletion region side microchannel–nanochannel interface (section 6.3), the enrichment region side nanochannel-microchannel interface (section 6.4) and, finally, the enrichment shock (section 6.5).

### 6. Focusing and stacking of anionic analytes

In this section we consider the behavior of anionic analytes in a system with concentration polarization due to a nanochannel with negatively charged walls. Here we extend the results of preliminary work we have published on this topic as well as the analysis of a recent paper by Plecis et al. Plecis et al. identifies four focusing regimes for anionic analytes and presents a computational model which captures phase boundaries based on surface charge, \( \sigma \), and applied potential. We here start in section 6.1 by deriving results for the behavior of anionic species using the analytical theory of Mani et al. Using an analytical model allows us to present a generalized regime map for arbitrary buffer and surface chemistry.

In sections 6.2–6.5, we compare both the analytical model based on Mani et al. and the computational results of Plecis et al. to published experimental results across a wide variety of conditions. We address separately each interface in a microchannel-nanochannel system with concentration polarization. Starting from the left side of Fig. 7 we will look at anionic analyte behavior on the depletion shock (section 6.2), the depletion region side microchannel–nanochannel interface (section 6.3), the enrichment region side nanochannel-microchannel interface (section 6.4) and, finally, the enrichment shock (section 6.5).

#### 6.1 Anionic analyte theory

To predict analyte behavior, we will look at the direction of transport of analyte anions in each region of the micro-nanochannel system relative to each interface in the system. As noted earlier, there are only four possible interfaces in a microchannel–nanochannel system with CP at which the total species drift velocity, \( \nu^{\text{tot}} = \nu^{\text{bulk}} - \nu^{\text{int}} \), changes. As shown in Fig. 7 these are the depletion shock, the microchannel–nanochannel interfaces and the enrichment shock. In this section to predict focusing versus stacking, we will examine \( \nu^{\text{int}} \) at each of these interfaces. Unlike cations, anions can, in the frame of reference of an interface moving at \( \nu^{\text{int}} \), migrate in opposite directions into a common focusing zone; so that

![Fig. 7 Summary of results for critical values of \( \nu^i \) for anionic analytes. For a given channel location and CP behavior (non-propagating, propagating \( \nu^i < 1 \), propagating \( \nu^i \geq 1 \)) if \( \nu^i \) is greater than \( \nu^i_{\text{crit}} \) then the ion will electromigrate upstream towards the anode, if \( \nu^i \) is less than \( \nu^i_{\text{crit}} \) then species \( \nu^i \) will advect downstream towards the cathode. The downstream direction (direction of bulk flow) and all four interfaces discussed in the text (depletion shock, microchannel–nanochannel interfaces, enrichment shock) are labeled in the microchannel–nanochannel schematic at the top of the figure. Details of this derivation are given in Appendix III.](image-url)
The nondimensional mobility of negatively charged analyte species in the enrichment region (the ratio of background electrolyte (cation) mobility to the electroosmotic mobility in the nanochannel). The dashed contours are critical values of the nondimensional mobility of negatively charged analyte species in the enrichment region ($\nu_i^{\text{crit}}$) or in the anode side reservoir region $\nu_i^{\text{crit}}$. If, for example, the nondimensional analyte mobility ($\nu_i^n$) is greater than the local $\nu_i^{\text{crit}}$, then the analyte will electromigrate upstream in the enrichment region. The solid line contours are critical values of analyte mobility in the reservoir regions ($\nu_i^{\text{crit}}$) for cases with no propagation, or contours of critical analyte mobility in the cathode side reservoir region ($\nu_i^{\text{crit}}$) for cases where CP propagates. These contours yield the mobility of an analyte which has zero total velocity in the reservoir region as a function of $c_0, h_i^n$ and $\nu_i^n$. The thick black line is the predicted phase boundary between propagating and non-propagating CP from eqn (5) and Fig. 4. Stacking and focusing predictions are discussed in the text below. Shown together with the theory are data from 8 studies. The predicted and observed focusing behaviors for each condition are discussed in the text and Table 2. Most of the results map as expected within our theoretical framework one exception is the experiments of Schoch et al. where enrichment side interface focusing was reported, but we predict stacking on the enrichment side interface.

As a preliminary note, $\nu_i^n$ is defined as the ratio of the analyte electrophoretic velocity to the bulk (electroosmotic) velocity inside the nanochannel ($\nu_i^n = \nu_i^{\text{eph}} / \nu_i^{\text{bulk}}$). Therefore, as presented in Appendix III, anolyte anions can never focus at this interface (since $\nu_i^{\text{eph}}$ cannot change sign). Experimental observations do show stacking at the enrichment shock, accompanied by a decrease in concentration in the near-nanochannel region. We defer detailed discussion of these results relative to experimental observations sections 6.2–6.5.

The interfaces created by the junctions of the microchannels and nanochannel have zero velocity, so that $V_i^{\text{int}} = 0$. Since, for a negatively charged channel system, anions electromigrate in the opposite direction of bulk flow, we will examine the ratio of the local electrophoretic velocity of the co-ionic species divided by the local bulk velocity, $U_i^{\text{ph}}/U_i^{\text{bulk}}$. Each of these velocities is here measured relative to the laboratory frame. If $U_i^{\text{ph}}/U_i^{\text{bulk}} > 1$, then the local electrophoretic velocity is higher than the local bulk velocity and the anionic species electromigrates upstream, against the direction of bulk flow. When $U_i^{\text{ph}}/U_i^{\text{bulk}} < 1$, the anionic species advects downstream in the direction of bulk flow. The ratio $U_i^{\text{ph}}/U_i^{\text{bulk}}$ in each region of the system, therefore, defines the critical value of the anion mobility, $\nu_i^{\text{crit}}$, at which the anion’s total velocity changes sign. We can then compare the non-dimensionnal analyte anion mobility, $\nu_i^n$, which is assumed to be constant and uniform throughout a given system, to the computed values of $\nu_i^{\text{crit}}$ in each region. If the analyte anion mobility, $\nu_i^n$, is greater than the critical value, $\nu_i^{\text{crit}}$, downstream of a particular interface, and less than the critical value upstream of the same interface, then that analyte anion will be transported inwards towards that interface and will focus. We examine this condition in more detail in sections 6.3 and 6.4.

At the enrichment region shock, $V_i^{\text{int}} = U_i^{\text{bulk}}$. Therefore, as presented in Appendix III, anolyte anions can never focus at this interface (since $U_i^{\text{eph}}$ cannot change sign). Experimental observations do show stacking at the enrichment shock, accompanied by a decrease in concentration in the near-nanochannel region. We defer detailed discussion of these results relative to experimental observations sections 6.2–6.5.

As a preliminary note, $\nu_i^n$ is defined as the ratio of the analyte electrophoretic velocity to the bulk (electroosmotic) velocity inside the nanochannel ($\nu_i^n = \nu_i^{\text{eph}} / \nu_i^{\text{bulk}}$). Therefore, as presented in Appendix III, anolyte anions can never focus at this interface (since $\nu_i^{\text{eph}}$ cannot change sign). Experimental observations do show stacking at the enrichment shock, accompanied by a decrease in concentration in the near-nanochannel region. We defer detailed discussion of these results relative to experimental observations sections 6.2–6.5.

At the depletion region shock $V_i^{\text{int}}$ is a known function of the background electrolyte and channel properties. In Appendix III we derive results for the critical nondimensional analyte mobilities, $\nu_i^{\text{crit}}$, at which ions will focus on the moving depletion shock. We examine focusing on the depletion shock in more detail in section 6.2.

** The enrichment shock is the only interface in such a system where $V_i^{\text{int}}$ is positive and nonzero. However, Mani et al. and Zangle et al. showed that this interface travels at the bulk velocity. Therefore, the total species velocity on either side of this interface will be $U_i^{\text{ph}}$ which has the same sign on either side of the enrichment shock.
electric field vary locally due to CP. For these zones the theory of Mani et al.\textsuperscript{11} can be employed to predict the relative strength of the electrophoretic to bulk velocities in each region of the micro-nanochannel system. Equating this ratio to unity yields the critical mobility in each zone in terms of key system parameters (channel height ratios, Dukhin number, and non-dimensional mobility of the co-in in the BGE). We defer the details of the analysis to Appendix III and present the results of critical mobilities in each zone in Fig. 7.

We again stress that the changes in $\nu_{crit}^i$ across each of the four interfaces in Fig. 7 indicate the existence and degree of a stacking or focusing situation due to local changes in electric fields and bulk flow velocities. For example, a decrease in the magnitude of $\nu_{crit}^i$ from region $e$ to $ce$ indicates slowing of anions as they move across the enrichment shock.\textsuperscript{††} This change of velocity is due to a change in electric field which, in turn, is caused by the concentration difference across this interface. (Note that since fluid mass is conserved, the bulk flow velocities on either side of the enrichment (or depletion) region shock in a constant area microchannel are the same, so a change in $\nu_{crit}^i$ reflects a change in local electric field.) This can lead to the case where an ion is transported downstream in one region relative to an interface (where $\nu_{crit}^i < \nu_{crit}^e$), and upstream in the next region relative to an interface (where $\nu_{crit}^i > \nu_{crit}^e$), hence focusing or stacking at the interface. Fig. 7 is therefore a general framework for predicting both stacking and focusing behavior in these systems.

The results for critical values of $\nu_{crit}^i$ are plotted in Fig. 8. Note that the critical value of $\nu_{crit}^i$ in the depletion region (region $d$) relative to either the depletion shock or the microchannel-nanochannel interface depends on the non-dimensional depletion region height, $h_{d}$. Over the domain of $c_{0i}^i, h_{d}^i$ and $\nu_{crit}^i$ represented in Fig. 8 and for the minimum reported value of $h_{d}^i h_{d}$ of 20,\textsuperscript{2}$\nu_{crit}^i \leq 0.05$. This critical value is lower than any computed $\nu_{crit}^i$ based on the literature surveyed here. In practice, this means that the electric field in the depletion region is high enough that all reported anionic species travel upstream faster than the depletion shock, with both $U_{jshock}^{eph}/U_{jbulk} > 1$ and $U_{jshock}^{eph}/U_{jbulk} > 1 - \nu_{shock}/U_{bulk}$. and, therefore, we do not plot values of $\nu_{crit}^i$ in Fig. 8. Table 1.

We illustrate the utility of Fig. 8 by considering an example case: a data point from Zangle et al.\textsuperscript{10} at $c_{0i}^i h_{d}^i = 0.3$ and $\nu_{crit}^i = 1.06$: In this case, experimental conditions (including the specific background electrolyte or buffer used) determine $c_{0i}^i$, $h_{d}^i$ and $\nu_{crit}^i$. We plot a point at these coordinates as shown in Fig. 7. Since the point is below the thick solid line, the model predicts propagating CP with large enrichment and depletion regions (a point above the line indicates non-propagating CP). We then either compute $\nu_{crit}^i, \nu_{crit}^e$ and $\nu_{crit}^i$ from the results in Fig. 7 or read their values from the $\nu_{crit}^i, \nu_{crit}^e$ and $\nu_{crit}^i$ contours of Fig. 8 to establish stacking or focusing situations.
Keep in mind that the critical value of \( n_i/C_3 \) in the nanochannel is unity, and the critical value of \( n_i/C_3 \) in the depletion region is typically lower than \( n_i/C_3 \) for most analytes. In this example, \( n_i/C_3 = 1.06 \), \( n_i/C_3 = 0.3 \) and \( n_i/C_3 = 1.06 \). Next, we compute the analyte nondimensional mobility \( n_i/C_3 \). In this case, we found \( n_i/C_3 = 0.34 \) for one of the analytes studied (see Table 2). Finally, stacking or focusing is possible only if the analyte slows down as it moves through an interface. Therefore, we identify interfaces where the critical analyte mobility is higher upstream than downstream (downstream direction defined as the direction of bulk flow in the system). In this example \( n_i/C_3 > n_i/C_3 \) and \( n_i/C_3 > n_i/C_3 \), so we predict an increase in analyte concentration at both the depletion and enrichment shocks. Next, we look at how the value of \( n_i/C_3 \) for our analyte of interest compares to the critical values in the system. In this case \( n_i/C_3 < n_i/C_3 \) and \( n_i/C_3 > n_i/C_3 \), therefore, this analyte is expected to stack strongly on the enrichment shock. This matches well with the experimental observation of strong enrichment shock stacking. This procedure was carried out for each of the experimental conditions in Fig. 8. The results of this analysis are summarized in Table 2 and the sections below. Appendix IV shows a supplementary figure where this example is worked out graphically.

In the following sections, we will consider possible focusing and stacking criteria at each interface shown in Fig. 7. We will also compare the model presented here (based in part on Mani et al.\(^{11}\)) to published experimental results. This comparison is summarized in Table 2.

### 6.2 Depletion shock

When CP propagates, focusing of analytes at the depletion shock occurs when the analyte mobility is low enough that analytes travel in the direction of bulk flow inside the reservoir region but high enough that they travel against bulk flow in the depletion region. This condition can be written as: \( n_i/C_3 > n_i/C_3 \) and \( n_i/C_3 > n_i/C_3 \). For all reported cases of \( n_i/C_3 \) from the literature, \( n_i/C_3 < n_i/C_3 \). This is because the concentration in the depletion region is very low, therefore, the electric field is very high. The high electric field in the depletion region causes all

---

**Table 2** Predicted and Observed Analyte Ion Behaviors. The parameters \( c_0, \nu^*_i, \nu^*_j \) and \( \nu^*_i \) are estimated based on literature data for zeta potential\(^{48-49}\) and mobility\(^{57,63-65}\)

<table>
<thead>
<tr>
<th>Reference</th>
<th>( c_0^* )</th>
<th>( \nu^*_i )</th>
<th>( \nu^*_j )</th>
<th>Predicted behavior</th>
<th>Observed behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pu, Q. et al. 2004(^{14})</td>
<td>0.03</td>
<td>0.22</td>
<td>0.37</td>
<td>Enrichment-side interface focusing</td>
<td>Enrichment-side interface focusing or enrichment shock stacking</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.25</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.28</td>
<td>0.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>0.31</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Datta, A. et al. 2006(^{18})</td>
<td>0.12</td>
<td>0.19</td>
<td>0.33</td>
<td>Enrichment-side interface focusing</td>
<td>Enrichment-side interface focusing</td>
</tr>
<tr>
<td>Schoch, R.B. et al. 2005(^{16})</td>
<td>0.07</td>
<td>0.42</td>
<td>0.12</td>
<td>Enrichment side interface stacking</td>
<td>Enrichment-side interface focusing</td>
</tr>
<tr>
<td></td>
<td>0.39</td>
<td>0.56</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plecis, A. et al. 2005(^{15})</td>
<td>0.06</td>
<td>1.43</td>
<td>0.72</td>
<td>Enrichment-side interface focusing</td>
<td>Enrichment-side interface focusing</td>
</tr>
<tr>
<td>Zangle, T.A. et al. 2007(^{10})</td>
<td>0.30</td>
<td>1.06</td>
<td>0.34</td>
<td>Enrichment shock stacking</td>
<td>Enrichment shock stacking</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>1.06</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.02</td>
<td>1.32</td>
<td>0.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dhopeshwarkar, R. et al. 2008(^{21})</td>
<td>0.43</td>
<td>4.66</td>
<td>1.77</td>
<td>Enrichment shock stacking</td>
<td>Enrichment shock stacking</td>
</tr>
<tr>
<td></td>
<td>0.43</td>
<td>4.66</td>
<td>2.33</td>
<td>No stacking</td>
<td></td>
</tr>
<tr>
<td>Hlushkou, D. et al. 2008(^{22})</td>
<td>0.43</td>
<td>4.66</td>
<td>0.59</td>
<td>Enrichment shock stacking</td>
<td>Enrichment shock stacking</td>
</tr>
<tr>
<td>Zhou, K. et al. 2008(^{26})</td>
<td>0.23</td>
<td>0.91</td>
<td>0.72</td>
<td>Depletion shock focusing</td>
<td>Depletion shock stacking/focusing</td>
</tr>
<tr>
<td></td>
<td>2.33</td>
<td>2.87</td>
<td>1.43</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
analytes to have a very high electrophoretic velocity here. Therefore, for all practical anions we know of in systems with typical microchannel heights \((c_\text{o}, h \gg 1)\), analyte ions will move upstream inside the depletion region (against the direction of bulk flow) faster than the depletion region shock velocity. Plecis et al.\(^9\) termed this regime anode cathode gradient focusing (ACGF).

Analytes which do not meet the focusing condition can still stack on the depletion shock as long as \(\nu_{i, c}^{\text{crit}} < \nu_{i, o}^{\text{crit}}\). For all CP cases examined here, this condition is met, and stacking at the depletion shock interface should occur. Physically, this condition means that anionic analyte ions traveling towards the depletion shock through the depletion region will slow down once they reach the shock, causing an increase in concentration.

The depletion shock is a strong shock\(^{11}\) and, therefore, has large changes in concentration over very short distances (see Fig. 3). These large concentration gradients in the background electrolyte create high electric field gradients. These field gradients, in turn, are very suitable for focusing of analytes to very high concentrations. We will discuss applications of focusing on a depletion shock in the last section of this review.

In an experiment\(^5\) with the same buffer and channel conditions as Zangle et al.\(^10\) we noted focusing on the depletion shock, as predicted by theory. Zhou et al.\(^26\) also noted focusing on the depletion shock, as expected by theory.

### 6.3 Depletion side microchannel–nanochannel interface

In the previous section we showed that anions can focus on the moving boundary of the depletion shock. Next, we will examine possible anion behaviors at the depletion-side microchannel–nanochannel interface. According to the theory presented in Fig. 7 and Fig. 8, focusing or stacking at the depletion side microchannel-nanochannel interface should not be possible if CP propagates. This is because the concentration in the depletion region is very low, therefore the electric field is very high, and co-ionic species travel very quickly upstream in this region. This corresponds to very low values of \(\nu_{i, d}^{\text{crit}}\) in the analytical theory outlined above. This focusing or stacking location corresponds to the anode stacking (AS) regime of Plecis et al.\(^9\).

As indicated schematically in Fig. 2, in cases where CP does not propagate, focusing or stacking is possible at this interface if \(\nu_{i, d}^{\text{crit}} < \nu_{i, p}^{\text{crit}}\). Focusing will occur if \(\nu_{i, d}^{\text{crit}} < \nu_{i, p}^{\text{crit}}\), but high enough to travel against bulk flow inside the nanochannel. This condition can be written as: \(\nu_{i, d}^{\text{crit}} < \nu_{i, p}^{\text{crit}} < \nu_{i, o}^{\text{crit}}\), or, since the critical analyte nondimensional mobility inside the nanochannel is always unity, we can write the condition for focusing on the enrichment side microchannel interface as

\[
\nu_{i, d}^{\text{crit}} < \nu_{i, p}^{\text{crit}} < 1
\]

(10)

For non-propagating CP there is no appreciable length of the enrichment region (any increase in background electrolyte concentration due to CP will be very localized). Therefore, we only need to consider the behavior of anions in the reservoir region. The condition for focusing at the enrichment side nanochannel interface for non-propagating CP is

\[
\nu_{i, d}^{\text{crit}} < \nu_{i, p}^{\text{crit}} < 1
\]

(11)

Finally, for propagating CP, stacking is possible at this interface if analyte ions have a higher electrophoretic velocity in the enrichment region than in the nanochannel. Therefore the stacking condition is

\[
\nu_{i, d}^{\text{crit}} < 1
\]

(12)

The stacking condition for non-propagating CP is

\[
\nu_{i, d}^{\text{crit}} < 1
\]

(13)

Pu et al.\(^14\) show behavior consistent with either focusing at the enrichment side nanochannel interface or stacking at an enrichment shock. The ambiguity is because, in the last time-series images reported, the regions of increased analyte concentration appear to be moving slightly away from the microchannel–nanochannel interface, though they have not propagated very far (see Fig. 6(f)-(j)). For all reported experimental cases our model predicts focusing at the enrichment side interface. Better comparison to experiments would require longer time series data. In related work, Datta et al.\(^16\) show results consistent with focusing on the enrichment side nanochannel interface and in agreement with our model.

Schoch, et al.\(^18\) also report focusing at the enrichment side nanochannel interface (Fig. 9), however, for their reported conditions, our model predicts either stacking on the enrichment side nanochannel interface or stacking on a propagating enrichment shock. This discrepancy may be due to the low

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applied potential in this experiment (10 V or less). A low applied electric field (and low Peclet number) may prevent the enrichment shock from propagating far enough to overcome diffusion in the vicinity of the pore. Another possible reason for this discrepancy may be our uncertainty in the mobility of GFP (we roughly estimated its mobility as $1 \times 10^{-9} \text{m}^2 \text{V}^{-1} \text{s}^{-1}$).\cite{Schoch2008} We also note that this Schoch paper discusses an increase of fluorescence concentration over time at a higher ionic strength. At this higher ionic strength, our model also predicts stacking at the enrichment side interface.

### 6.5 Enrichment shock

Here, we consider the final interface shown in Fig. 7, the enrichment shock. The model presented in section 6.2 predicts that anionic species can stack at the interface between the enrichment region and the reservoir region during propagating CP. This behavior is accurately described as stacking because in a frame of reference moving with the enrichment shock, the anionic total species velocity does not change its sign across the shock.\cite{Hlushkou2008} Physically, the enrichment shock propagates at the bulk velocity,\cite{Hlushkou2008} so in a frame moving with the enrichment shock, the only component of the species velocity is due to electrophoresis. The electrophoretic velocity is always negative, and therefore, does not cross zero at the shock front. It is possible, however, for anions to have a net inward velocity (towards the enrichment shock) in the lab frame.

When CP propagates, analytes move inwards on either side of the enrichment shock if their mobility is low enough to travel in the direction of bulk flow inside the enrichment region but high enough to travel against bulk flow in the reservoir region. This condition can be written as: $v_{\text{i,le}}^{\text{crit}} < v_i < v_{\text{i,le}}^{\text{crit}}$. Analytes which do not meet this condition can still stack on the enrichment shock provided $v_{\text{i,le}}^{\text{crit}} < v_{\text{i,le}}^{\text{crit}}$. If this second condition is met, the shock will slow down in the enrichment region, causing an increase in concentration.

Enrichment shock stacking was described analytically by Zangle et al.\cite{Zangle2010} and computationally by Plecis et al.,\cite{Plecis2021} who called this type of stacking cathodic counter gradient focusing, or CCGF. Zangle et al.,\cite{Zangle2010} presented results for two specific buffer cases in terms of a Dukhin number and the nondimensional mobility of the focusing species and correctly predicted the observed stacking behavior. The latter paper also presented experiments in which the enrichment region shock was used to stack and separate ions on the basis of their electrophoretic mobilities. This is possible because, in propagating CP, the enrichment shock is a weak shock\cite{Hlushkou2008} which creates an order 100 μm long region with gradients in concentration and electric field. In contrast, the strong shock of the depletion region shock is typically much shorter (roughly on the order of the microchannel width), and, therefore, not typically appropriate for separation. The length of the enrichment shock is typically much longer than the ~10 μm peak widths of focused species; and so species visibly separate.\cite{Zangle2010} To date, we have not been able to (and have not seen in published work) significant separation resolution achieved using enrichment shocks. We hypothesize that the relatively low ratio of effective electric fields across these interfaces simply yield low peak capacity; further, preconcentration is limited to order 100×.\cite{Zangle2010, Dhopeshwarkar2021}

Papers by Hlushkou et al.\cite{Hlushkou2008} and Dhopeshwarkar et al.\cite{Dhopeshwarkar2021} each presented an enrichment region stacking scheme along with a supporting computational model. A sample experimental result is shown in Fig. 10. As summarized in Table 2, these experimental results agreed well with the prediction of enrichment shock stacking.

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**Fig. 9** Visualization of green fluorescent protein (GFP) focusing at the enrichment side nanochannel interface. The model presented here predicts that this analyte should focus on a propagating enrichment shock, not at this interface. We hypothesize the discrepancy may be due to the low field values used or to our uncertainty in GFP mobility at these conditions. Reprinted with permission from Schoch et al.\cite{Schoch2008}

**Fig. 10** Results from Hlushkou et al.\cite{Hlushkou2008} showing stacking of an anionic species on an enrichment shock. (a) shows the channel before a potential was applied. At 960 s after the voltage was applied (b), an enriched fluorescent sample zone has formed and moved away from the interface between the microchannel and the anionic, nanoporous HEMA hydrogel. These dynamics are characteristic of co-ionic species stacking on an enrichment region shock. Theory (both the theory presented here and that of Hlushkou et al.\cite{Hlushkou2008} and Dhopeshwarkar et al.\cite{Dhopeshwarkar2021}) suggests that this occurs when the fluorescent species electrophoretic velocity is equal to the bulk velocity at the CP enrichment shock (in the lab frame). Reprinted with permission from Hlushkou et al.\cite{Hlushkou2008} Copyright 2008 Royal Society of Chemistry.
7. Applications

In this section, we present several example applications of CP in devices which are tools for analysis of chemical and biological species or fluid pumping. The main application of CP has been for on-chip sample preconcentration. In a 2005 paper, Wang et al. used current through a nanochannel to create a CP depletion zone, and then focus protein on the boundary between the depletion zone and a region of the microchannel at the reservoir concentration. This technique requires that the preconcentrator use a different geometry than the serial microchannel-nanochannel-microchannel geometry considered here. The Wang et al. device has two independently-controlled, anode-side channels so that an electroosmotic flow can be directed perpendicular to the axis of the nanopore. We hypothesize that, with this geometry, the depletion shock can be immobilized by adjusting the ratio of current across the two anode side channels (which controls bulk flow rate) to the current through the nanochannel (which controls the depletion region shock velocity). The Wang study reported $10^6$-$10^7$ fold increase in concentration of a target protein, GFP (although we believe that at least the higher preconcentration value is an overestimate). In a similar study at University of Michigan, Kim et al. used irreversible PDMS-glass bonding as a different method to create charged nanofluidic slits and reported $10^5$-$10^6$-fold increase in fluorescence intensity.

Since the first demonstration of basic microfluidic-nanofluidic CP preconcentrators, there have been a number of other applications of CP-based sample preconcentrators. For example, Wang et al. reported increased range and 500-fold improvement in sensitivity of immuno detection using immobilized, antibody labeled beads. In that study, the authors immobilized labeled beads and stopped the depletion shock at the same location inside the anode-side microchannel, as shown in Fig. 11(a). The device then preconcentrated antigens using focusing on the strong electric field gradients of the depletion shock, which increased the rate of binding to antibody labeled beads.

Kim et al. used vortices generated at a microchannel-nanochannel interface to create a mixing device. This device consisted of a set of parallel nanochannels connected to a U-shaped microchannel and is shown in Fig. 11(b). Here, the strong electric field gradients at the depletion shock boundary are used to create strong vortices which mix the two fluid streams.

Finally, the nonlinear behavior of electroosmotic flow in the depletion region can be used to create an induced zeta potential which is proportional to the applied electric field, a phenomena called non-equilibrium electroosmosis, or electroosmosis of the second kind. In flows of this kind, the total fluid velocity is proportional to electric field squared. This is in contrast to equilibrium electroosmosis where zeta potential is static and fluid velocity is directly proportional to the applied electric field. This has lead to work on using CP effects to pump fluids. In particular, in a device using nonequilibrium EOF Kim et al. reported a 5-fold increase in volumetric flow rate relative to a similar device using equilibrium electroosmosis.

8. Summary

We presented a summary of current theory describing the regimes and dynamics associated with CP in systems with microchannel-nanochannel interfaces. We summarized results from 56 sets of published experiments and found good agreement to published theory in delineating the regimes which separate propagating versus non-propagating CP. This comparison shows that the parameters which govern CP in a microchannel-nanochannel system are an inverse Dukhin number and the ratio of background electrolyte cation mobility to the electroosmotic mobility inside the nanochannel.

![Fig. 11](image_url) Selected applications. (a) Nanofluidic preconcentrator used to increase sensitivity and dynamic range of a bead-based immunoassay. Antibody-labeled beads are loaded into the microchannel and held in place adjacent to the anode side of the nanochannels used for preconcentration (top image). When an electric field is applied, a zone of focused analyte forms on top of the immobilized beads, increasing the antibody-antigen binding rate. (b) CP-based mixing device. When an electric field is applied from the sample wells (at bottom, labeled with voltage, $V$) to the outlet well (labeled Gnd) the high electric field across the nanochannel creates a strong depletion region. Instability at the boundary of this depletion region mixes the two sample streams as shown in the images taken approximately 150 ms apart in the inset (bottom right). Adapted with permission from Wang et al. and Kim et al. Copyright 2008 Royal Society of Chemistry.
We then used the results of Mani et al.\textsuperscript{11} to predict the behavior of cationic and anionic analytes in a micro-channel–nanochannel system with negative wall charge. This included an extension of this theory to predict the conditions required for focusing or stacking of analytes in systems with CP. We predict stacking and/or focusing at four interfaces which can occur in a microchannel–nanochannel–microchannel system. We presented a comparison of this theory to published experimental data. Again we found good agreement between the reported cases from literature and the extension of the experimental data. Again we found good agreement between the theory we summarize and validate in view of the applied potential or electric field.

Overall, the theory we summarize and validate in view of experimental data can be used to understand and interpret the effects of CP on analyte transport in systems containing both micro- and nanochannels.

9.1 Appendix I. Derivation of convective current through a nanochannel

In this appendix we derive a simple expression for convective current due to charge transport by electroosmotic flow through a thin channel. While a more general derivation is presented in Levine et al.,\textsuperscript{61} we here consider an extreme case in which ions associated with the EDL are the only ions in the channel and the channel is very thin \((Fz_{\text{ref}}/\sigma \ll 1)\). Under these conditions the wall-shielding charge will have a uniform concentration equal to: \(c = -2\sigma/zFh\). The solution to the Poisson–Boltzmann equation will yield the following parabolic potential:

\[
\psi - \zeta = -\frac{\sigma}{zFh} y(h - y) \tag{14}
\]

From eqn (1) the convective current is:

\[
I_{\text{conv}} = w \int_0^h FzE U^{\text{bulk}} \, dy \tag{15}
\]

where, from Levine et al.\textsuperscript{61} \(U^{\text{bulk}} = \varepsilon \varepsilon_0 (\psi - \zeta) \eta\). Substituting expressions for \(c\) and \(U^{\text{bulk}}\) and using eqn (14) yields

\[
I_{\text{conv}} = \frac{w \sigma^2 E}{3\eta}. \tag{16}
\]

9.2 Appendix II. Derivation of stacking ratio for cationic analytes

As shown in Fig. 5, the microchannel–nanochannel–microchannel system can in general be divided into the following five distinct zones: 1. the undisturbed left-side microchannel, with the same concentration as in the reservoir; 2. the depletion region (if CP propagates); 3. the nanochannel; 4. the enrichment zone (if CP propagates), and 5. the undisturbed right-side microchannel. These five zones are separated by four boundaries which present possible interfaces for stacking or focusing of a cationic analyte. We assume that the cationic analyte is released from the anode-side reservoir into the channel system. In addition, we assume that the analyte is released after the nanochannel establishes its equilibrium condition. Furthermore, we consider low concentration cationic analytes compared to the background electrolyte (BGE), so that it cannot affect the axial electric field. Also, in this derivation and the anionic analyte derivation in Appendix III, following the analysis of Mani et al.\textsuperscript{11} we will assume high \(PeL/h\) (i.e., high product of Peclet number based on channel height and the ratio of channel length to nanochannel height) and so will not present results as a function of the applied potential or electric field.

We first present the results of the simple theory of Mani et al.\textsuperscript{11} for the BGE, which determines translocation regimes in the system. Then we use their theory as a foundation to describe the effects of CP on low concentration cationic analytes.

The simplest model presented by Mani et al.\textsuperscript{11} suggests that the bulk concentration of binary electrolyte in a channel system satisfies the following one-dimensional differential equation:

\[
\frac{\partial}{\partial t} (h^* c_0^*) + \frac{\partial}{\partial x} \left( c_0^* + I^* \frac{c_0^*}{c_0^* + 1} \right) = \frac{1}{Pe} \frac{\partial}{\partial x} \left( h^* \frac{\partial c_0^*}{\partial x} \right) \tag{17}
\]

where \(h^*\) is the height of the channel, non-dimensionalized by a nanoscale reference height, \(h_{\text{ref}}\). \(c_0^*\) is the nondimensional bulk concentration using the wall surface-charge density, \(\sigma\), which is assumed to be constant and uniform in this model:

\[
c_0^* = \frac{\nu_1 z_1 - \nu_2 z_2}{2\nu_1 z_1} \frac{Fz_{\text{ref}} \varepsilon_0}{\sigma Q} \tag{18}
\]

With this notation, \(c_0^* h^*\) is the local inverse Dukhin number. \(I^*\) is the nondimensional current through the system relative to the bulk flow rate:

\[
I^* = \frac{\nu_2 z_2}{2\nu_1 z_1} \frac{h_{\text{ref}}}{\sigma Q} \tag{19}
\]

where \(Q\) is the bulk flow rate. The axial coordinate and time are nondimensionalized by \(h_{\text{ref}}\) and \(h_{\text{ref}}/Q\), respectively.

Eqn (17) is a nonlinear advection-diffusion equation for evolution of the BGE, \(c_0^*\). The non-dimensional advection flux has two terms: \(c_0^*\) which represents linear advection by the bulk flow rate, and \(I^*/(h^* c_0^* + 1)\) which represents exchanges between the bulk and EDL. The axial electric field in the system is the ratio of the current and local conductivity:

\[
E = \frac{I}{Fz_1 \left[ F(\nu_1 z_1 - \nu_2 z_2) h_{\text{ref}} - 2\nu_1 z_1 \sigma \right]} \tag{20}
\]

The total conductivity appeared in the denominator of eqn (20) consists of bulk effects of co- and counter-ions and the EDL conductivity. Note that \(c_0\) is the bulk concentration of counter-ions in the BGE. From net neutrality, the bulk concentration of co-ion would be \(z_1 c_0/z_2\), which would lead to the expression in the denominator of eqn (20) for conductivity.
An analyte ion with mobility $\nu_i$ and valence number of $z_i$ would experience an electromigration velocity equal to $\nu_i z_i FE h$.

Using eqn (20) and the nondimensionalization procedure from Mani et al.\textsuperscript{11} we arrive at the following expression for nondimensional electromigration velocity of the analyte:

$$\frac{\text{electromigration velocity}}{\text{bulk velocity}} = \frac{\nu_i z_i FE h}{Q} = \frac{\nu_i z_i}{h^*} \frac{h^*}{h c_0^{*}} + 1.$$  \hfill (21)

In the system considered here, the current and flow rate are not independent of each other. The current drives the electroosmotic flow in the nanochannel since the nanochannel has the dominant hydraulic resistance.

Gradients created at the microchannel–nanochannel interfaces are independent of each other. The current drives the electroosmotic flow through the system and the internal pressure in the left-side-microchannel is:

$$I^* = \frac{\nu_i^*}{h_{0}^{*}} (h_{n}^{*} c_{0,n}^{*} + 1).$$  \hfill (22)

As described earlier, $\nu_i^*$ is the nondimensional mobility of coions to the wall charge. It is with the assumption of eqn (22) that the entire BGE system can be parameterized by two parameters, $\nu_i^*$ and inverse Dukhin number in the nanochannel, $c_{0,n}^{*}, h_{n}^{*}$. Substituting eqn (22) into eqn (21) yields:

$$\frac{\nu_i z_i FE h}{Q} = \frac{\nu_i^* h^n (h_{n}^{*} c_{0,n}^{*} + 1)}{h^n (h c_0^{*}) + 1},$$  \hfill (23)

where $\nu_i^*$ is the nondimensional mobility of the analyte specie. Eqn (23) can be used to determine the global translocation of ions at each zone in the channel system. In this appendix, we use this equation to analyze stacking of cationic analytes for different CP regimes.

### 9.2.1 Non-propagating CP

If CP does not propagate, the channel system will present three zones as shown in Fig. 2c: 1. the left-side microchannel, with uniform concentration the same as in the reservoir; 2. the nanochannel; and 3. the right-side microchannel with uniform concentration the same as in the reservoir. Stacking is possible at either of the two interfaces between the microchannels and the nanochannel. For this case, the equilibrium BGE concentration in the nanochannel would be lower than the reservoir’s concentration due to significant flux carried by the EDL\textsuperscript{11}

$$c_{0,n}^{*} = c_{0,r} - \frac{\nu_i}{h^n}.$$  \hfill (24)

If one releases a cation analyte specie, $i$, into the left reservoir with concentration $c_{i,r}$, the analyte will travel through the left-side microchannel. Ignoring the EDL effects (in this microchannel with $c_{0,n}^{*}, h_{n}^{*} \gg 1$), the flux of the analyte in the left-side-microchannel is:

$$f_{i, L} = c_{i,r} (Q + \nu_i z_i FE h_n)$$  \hfill (25)

In general, the concentration of the analyte in the nanochannel, $c_{i,n}$ would be different from its concentration in the microchannels. The flux of the analyte in the nanochannel can be written as:

$$f_{i, n} = c_{i,n} (Q + \nu_i z_i FE h_n) + f_{i, EDL}^{EDL}$$  \hfill (26)

where $f_{i, EDL}^{EDL}$ is the flux of species $i$ in the nanochannel (region $n$) through the EDL. We note that $c_{i,n}$ represents only the bulk concentration of the analyte; therefore, to account for the total flux, the EDL contribution should be added. To estimate the portion of the analyte in the EDL here we assume that the analyte is distributed between the bulk and the EDL with the same proportion as in the cationic BGE. This assumption is only valid when the analyte has the same valence number as the cation in the BGE and thus satisfies the same Boltzmann distribution in the wall-normal direction:

$$\frac{\int_{0}^{h} c_{i, EDL} dy}{\int_{0}^{h} c_{i, Bulk} dy} = \frac{\int_{0}^{h} c_{i, EDL} dy}{\int_{0}^{h} c_{i, Bulk} dy} = \frac{-2 \sigma_F z_i}{c_{0}^{*} h^n}$$  \hfill (27)

The second equality in eqn (27) is consistent with the simple theory of Mani et al.,\textsuperscript{11} assuming that only the excess concentration of cations contributes to formation of the EDLs. Following the nondimensionalization procedure of Mani et al.,\textsuperscript{11} the right-hand-side of eqn (27) can be written in terms of nondimensional quantities as:

$$\frac{\int_{0}^{h} c_{i, EDL} dy}{\int_{0}^{h} c_{i, Bulk} dy} = \frac{1 - \nu_i z_i}{c_{0}^{*} h^n}$$  \hfill (28)

To simplify the algebraic manipulations, in the next steps we here assume that the BGE is a symmetric electrolyte (i.e., mobility and valence for the anionic BGE species equal the mobility and valence for the cationic BGE species). With this simplification, the numerator of eqn (28) reduces to a value of 2 and we arrive at a simple expression for the EDL to bulk contribution of analyte cations in terms Dukhin number based on the BGE concentration:

$$\frac{\int_{0}^{h} c_{i, EDL} dy}{\int_{0}^{h} c_{i, Bulk} dy} = \frac{2}{c_{0}^{*} h^n}$$  \hfill (29)

Using this expression we can write the EDL flux in eqn (26) as a simple correction to the bulk flux.

$$f_{i, n} = c_{i,r} [Q + \nu_i z_i FE h_n (1 + 2/c_{0,n}^{*} h_n^{*})]$$  \hfill (30)

We note that consistent to the simple model of Mani et al.,\textsuperscript{11} we here assumed that ions in the EDLs are only transported via electromigration. The correction factor, $(1 + 2/c_{0,n}^{*} h_n^{*})$ is simply the total to bulk concentration ratio for counterions. Due to conservation of cationic analyte $i$, the flux of the analyte in the microchannel is equal to that in the nanochannel. By equating the expressions in eqn (25) and (30) one can obtain an expression for the stacking ratio of the analyte:

$$\frac{c_{i,n}}{c_{i,r}} = \frac{Q + \nu_i z_i FE h_n}{Q + \nu_i z_i FE h_n (1 + 2/c_{0,n}^{*} h_n^{*})}$$  \hfill (31)
With this form stacking ratio is defined in terms of analyte concentrations in the bulk. A more useful quantity, however, is the stacking ratio in terms of the cross-sectional area-averaged concentrations (height-averaged concentrations in this case). This requires us to include the EDL contribution in the definition of stacking ratio. In our notation we use $S_{i,n/r}$ to refer to the area-averaged stacking ratio from region $r$ to region $n$ for analyte specie $i$.

$$S_{i,n/r} = \frac{c_{i,n} (1 + 2/c_{0,n} h_n^*)}{c_{i,r}} = \frac{1 + \nu_i z_i F E a h_i / Q}{1/(1 + 2/c_{0,n} h_n^*) + \nu_i z_i F E a h_n / Q}$$

(32)

In the next step we can write the stacking ratio in terms of system parameters, $c_{i,r}^*, h_n^*$, $\nu_i^*$, and $\nu_i^*$. This can be simple done by substituting eqn (23) into (32) and then substituting eqn (24) into the resulting expression. Ignoring the EDL contribution of the microchannel, when evaluating eqn (23) for region $r$ (ie. ignoring 1 relative to $h_i^* c_{i,r}^*$) leads to the following expression for $S_{i,n/r}$:

$$S_{i,n/r} = c_{i,r}^* h_n^* + \nu_i^* (c_{i,r}^* h_n^* - \nu_i^*) + c_{i,n}^* h_n^* - \nu_i^* + 2$$

$$c_{i,r}^* h_n^* - \nu_i^* + 2$$

$$c_{i,r}^* h_n^*$$

(33)

It will be useful to write this expression in terms of the stacking ratio of the background electrolyte, $S_{0,n/r}$ (here in terms of background cations). Having $S_{0,n/r} = (1 + 2/c_{0,n} h_n^*) c_{0,n} / c_{0,r}$ (see definition in eqn (32), eqn (33) can be written in the following form:

$$S_{i,n/r} = S_{0,n/r} 1 - \nu_i^* c_{i,r}^* h_n^* + \nu_i^* S_{0,n/r} / 1 - \nu_i^* c_{i,r}^* h_n^* + \nu_i^* S_{0,n/r}$$

(34)

where $S_{0,n/r}$ is equal to $(c_{i,r}^* h_n^* - \nu_i^* + 2)/(c_{i,r}^* h_n^*)$. We note that since this expression is for non-propagating CP the denominator of eqn (34) will never equal zero. Also, eqn (34) indicates that if the analyte is slower than the BGE ($\nu_i^* < \nu_i^*$) then the stacking ratio of the analyte will be higher than the stacking ratio of the background electrolyte cation.

9.2.2 Propagating CP, $\nu_i^* \geq 1$

For this case we will start by computing the stacking ratio across the depletion front. We consider a control volume around the depletion shock moving with the shock velocity. The flux of analyte species from region $r$ into this control volume is:

$$f_{i,r} = c_{i,r} Q - \nu_i z_i F E a h_i$$

(35)

The depletion shock velocity given by the theory of Mani et al. is equal to

$$\nu_i z_i F E a h_i / Q = 1 - 2\nu_i^* - 1$$

(36)

The flux of analyte species from the control volume into the depletion zone can be written in a similar fashion. However, note that since the bulk concentration is very low in the depletion zone, the EDL contribution is not negligible in this zone and should be considered in the expression for the flux. Following a similar method that lead to eqn (30), we can write:

$$f_{i,d} = c_{i,d} [Q - \nu_i z_i F E a (h_d + (1 + 2/c_{0,d} h_d^*))]$$

(37)

Since the background electrolyte has a very low concentration in the depletion region, current conservation demands an extremely high electric field in this region. Therefore the flux associated with the electric field has the dominant contribution in the flux term and thus, $f_{i,d}$ can be approximated as:

$$f_{i,d} \simeq c_{i,d} (1 + 2/c_{0,d} h_d^*)$$

(38)

By equating the flux expressions in eqn (38) and (35) and using eqn (36), one can obtain an expression for the stacking ratio from the reservoir to the depletion zone:

$$S_{i,d/r} = \frac{c_{i,d} (1 + 2/c_{0,d} h_d^*)}{c_{i,r}^* h_n^* + \nu_i^* (c_{i,d}^* h_n^* - \nu_i^*) + c_{i,n}^* h_n^* - \nu_i^* + 2}$$

$$\frac{c_{i,d}^* h_n^* - \nu_i^* + 2}{c_{i,d}^* h_n^*}$$

(39)

Here, we used eqn (23) to express the ratios of electromigration fluxes to the advective flux. Mani et al. found the following expressions for the BGE concentrations in the depletion zone and the nanochannel:

$$c_{0,d} h_d^* = (\nu_i^* - 1)^2 / 2\nu_i^* - 1$$

(40)

Using these expressions in eqn (39) and noting $c_{0,d} h_d^* \gg 1$ for a typical microchannel leads to

$$S_{i,d/r} = \nu_i^* (2\nu_i^* - 1 + \nu_i^* \nu_i^*) / 2\nu_i^* - 1$$

(41)

Rewriting this equation in terms of stacking ratio of the BGE leads to

$$S_{i,d/r} = S_{0,d/r} \left[ \frac{\nu_i^* (2\nu_i^* - 1 + \nu_i^* \nu_i^*)}{2\nu_i^* - 1 + \nu_i^* \nu_i^*} \right]$$

(42)

which is a monotonically decreasing function of $\nu_i^*$. Next, we derive an expression for the stacking between the depletion zone and the nanochannel. Since the electromigration flux is the dominant term in the depletion zone, eqn (38) is still a good approximation for the flux in the depletion zone (although this equation was originally derived for a moving control surface). The flux in the nanochannel is already derived in eqn (30) except here the value of the nanochannel concentration should be replaced from eqn (40).

$$S_{i,n/d} = \frac{c_{i,n} (1 + 2/c_{0,n} h_n^*)}{c_{i,d} (1 + 2/c_{0,n} h_d^*)} = \frac{Q}{Q/(1 + 2/c_{0,n} h_n^*) + \nu_i z_i F E a h_n}$$

(43)
Using eqn (23) and (40) into eqn (43) leads to

\[ S_{i,n/d} = \frac{h_d}{h_0} \left( \frac{\nu_i^2}{\nu_i^2 + 1} \right) \frac{\nu_i^2 - 1 + \nu_i \nu_i^2}{\nu_i^2 - 1 + \nu_i \nu_i^2} (44) \]

Rewriting this equation in terms of stacking ratio of the BGE leads to

\[ S_{i,n/d} = S_{0,n/d} \left( \frac{\nu_i^2 - 1 + \nu_i \nu_i^2}{\nu_i^2 - 1 + \nu_i \nu_i^2} \right) (45) \]

We can obtain a direct stacking ratio from the reservoir to the nanochannel by multiplying the expressions in eqn (45) and (42):

\[ S_{i,n/r} = \frac{2\nu_i^2 - 1 + \nu_i \nu_i^2}{\nu_i^2 + \nu_i^2 - 1 + \nu_i \nu_i^2} (46) \]

We can also write the stacking ratio from the reservoir to the nanochannel without relating it to the BGE concentration as:

\[ S_{i,n/r} = \frac{(2\nu_i^2 - 1 + \nu_i \nu_i^2)(\nu_i^2 + 1)}{c_{0_r}^* h_n^* \left( \nu_i^2 - 1 + \nu_i \nu_i^2 \right)} (47) \]

Next, we derive an expression for the stacking ratio from the nanochannel to the enrichment zone. In the enrichment zone the EDL contribution to the flux can be neglected and the total flux can be written as:

\[ f_{i,e} = c_{i,e} \left( Q + \nu_i \varepsilon F E R_i h_e \right) (48) \]

Equating this flux to the flux associated with the nanochannel leads to an expression for the stacking ratio from nanochannel to the enrichment zone:

\[ S_{i,e/n} = \frac{1 + 2\nu_i^2}{1 + \nu_i \nu_i^2} \left( \frac{c_{0,e}^* h_n^*}{c_{0,n}^* h_n^*} + \nu_i^2 \right) (49) \]

Here, we used eqn (23) to express the ratios of electromigration fluxes to the advective flux. From Mani et al., the BGE concentration in the enrichment zone for this case is:

\[ c_{0,e}^* h_n^* = 2\nu_i^2 - 1 (50) \]

Using this expression together with eqn (40) in eqn (49) and noting \( c_{0,e}^* h_n^* \gg 1 \) for typical microchannels leads to:

\[ S_{i,e/n} = \left( \frac{2\nu_i^2 - 1}{\nu_i^2 + 1} \right) \left( \frac{\nu_i^2 - 1 + \nu_i \nu_i^2}{\nu_i^2 - 1 + \nu_i \nu_i^2} \right) (51) \]

Rewriting eqn (51) in terms of the BGE stacking ratio leads to:

\[ S_{i,e/n} = S_{0,e/n} \left( \frac{\nu_i^2 + \nu_i^2 - 1 + \nu_i \nu_i^2}{2\nu_i^2 - 1 + \nu_i \nu_i^2} \right) (52) \]

By combining eqn (52) and (46) one can see:

\[ S_{i,e/r} = S_{0,e/r} (53) \]

We can also write:

\[ S_{i,e/r} = \frac{2\nu_i^2 - 1}{c_{0,e}^* h_n^*} (54) \]

Finally we show that the concentration of the analyte in the cathode-side \( r \) region is the same as that in the anode-side \( r \) region. Here, we will temporarily use subscript \( rc \) for the cathode-side microchannel to distinguish it from the anode-side zone. We consider a control volume moving with the enrichment front and write the flux balance equation between the two sides of the front. Mani et al., and Zangle et al., showed that the enrichment shock advects at the bulk velocity. Furthermore, EDL contribution to the flux terms is negligible on either sides of the enrichment shock. Therefore, the only term contributing to the flux through the control surfaces is the electromigration of the bulk concentration, henceforth the flux balance equation leads to:

\[ S_{i,rc/e} = \frac{c_{i,rc}}{c_{i,e}} = \frac{\nu_i \varepsilon F E R_i h_e}{E_e} = \frac{E_e}{E_{rec}} = \frac{c_{0,rc}}{c_{0,e}} = S_{0,rc/e} (55) \]

Combining eqn (53) and (55) leads to:

\[ S_{i,rc/e} = S_{0,rc/e} = 1 (56) \]

The second equality is due to the fact that the anode-side reservoir has the same BGE concentration as in the cathode-side reservoir.

9.2.3 Propagating CP, \( \nu_i < 1 \)

The procedure of solving the problem in this case is exactly the same as what we presented in section 9.2.2. The only difference is that the BGE concentrations are different in this case and the following values from solution by Mani et al., should be substituted into the expressions:

\[ c_{0,d} = c_{0,n} = 0, c_{0,e} h_n^* = \nu_i^2 (57) \]

For this case the depletion region shock velocity is given by:

\[ \frac{V_{shock} h_e}{Q} = 1 - \frac{\nu_i^2}{c_{0,e} h_n^*} (58) \]

In this case in regions \( d \) and \( n \) the only term contributing to the fluxes is the EDL transport and the bulk term is zero. We avoid repeating the algebraic procedures here and only present the results of the analysis.

\[ S_{i,d/r} = \frac{\nu_i^2 + \nu_i^2}{h_{e,c_{0,d},r} \nu_i^2} = S_{0,d/r} \left( \frac{\nu_i^2 + \nu_i^2}{2\nu_i^2} \right) (59) \]

For the depletion region we obtain:

\[ S_{i,n/d} = \frac{h_d}{h_0} = S_{0,n/d} (60) \]

and combining eqn (60) and (59):

\[ S_{i,n/r} = \frac{\nu_i^2 + \nu_i^2}{h_n^* \nu_i^2 \nu_i^2} (61) \]

Finally, for the enrichment region

\[ S_{i,e/n} = \frac{\nu_i^2 \nu_i^2}{\nu_i^2 + \nu_i^2} = \frac{S_{0,e/n} 2\nu_i^2}{\nu_i^2 + \nu_i^2} (62) \]

where, using eqn (61), we obtain:

\[ S_{i,e/r} = \frac{\nu_i^2}{h_d^* c_{0,e}^*} (63) \]
9.3 Appendix III. Analysis of focusing for anionic analytes

The objective of this appendix is to derive expressions for the critical mobility of anionic analytes in a microchannel–nanochannel–microchannel system with CP. If an analyte’s mobility is higher than the local critical value, it will travel against the flow direction, whereas sub-critical analytes will have net transport in the same direction as the flow. Focusing is possible in regions where the analyte velocity field is convergent. In other words, focusing is possible where the analyte is super-critical downstream of the focusing zone and sub-critical upstream. Therefore, knowing critical mobilities at different zones of the channel system allows us to identify possible focusing zones.

It should be noted that the analysis of anion transport is less complex compared to that for cations since anions are mostly transported through the bulk and have a relatively low concentration in the EDLs. Therefore, for the purposes of this analysis, we will assume that an anionic analyte, i, is transported due to electromigration in the bulk in one direction and advection due to bulk flow in the opposite direction. The ratio of the electromigration velocity to the bulk flow velocity is, therefore:

\[
\frac{U_i^{\text{eph}}}{U_{\text{bulk}}} = \frac{v_i z_i F \varepsilon H}{Q}
\]

In Appendix II we derived an expression for this ratio in the nondimensional form. Using eqn (23) in eqn (64) we have:

\[
\frac{U_i^{\text{eph}}}{U_{\text{bulk}}} = v_i \left( \frac{c_i^{\text{d},+} h_i^* + 1}{c_i^{\text{d},+} h_i^* + 1} \right)
\]

where \( v_i \) is nondimensionalized such that it is always positive (\( v_i = \frac{v_i z_i F \varepsilon H}{\varepsilon H} \)). At either side of a stationary interface in the channel system, the critical analyte mobility can be computed by equating the right-hand side of eqn (65) to unity:

\[
v_i^{\text{crit}} = \frac{h_i^* (c_i^{\text{d},+} h_i^* + 1)}{c_i^{\text{d},+} h_i^* + 1}
\]

As discussed earlier, focusing will occur at a moving interface when the total species velocity in a frame moving with the interface, \( U_i^{\text{int}} / U_{\text{bulk}} - V_{\text{int}} \), changes sign. Therefore, at a moving interface, the critical analyte mobility can be computed by equating the right hand side of eqn (65) to one minus the interface velocity normalized by the bulk velocity:

\[
v_i^{\text{crit}} = \left( 1 - \frac{U_i^{\text{int}}}{U_{\text{bulk}}} \right) \left( \frac{c_i^{\text{d},+} h_i^* + 1}{c_i^{\text{d},+} h_i^* + 1} \right)
\]

Note, however, that the enrichment shock velocity is equal to the bulk velocity \( v_i^{\text{crit},+} \) and so at this interface analyte anions can never focus (on both sides of this interface, \( v_i > v_i^{\text{crit},-} \), and so the analyte anion total velocity can never change sign). However, experimental observations have shown that, under some conditions anionic species can be transported inwards towards the enrichment shock in the lab frame.\(^{21,22,9,10}\) Therefore, on either side of the enrichment shock we will look at the total species velocity in the lab frame and define \( v_i^{\text{crit}} \) using eqn (66).

For different CP regimes, Mani et al.\(^ {11} \) derived expressions for concentration in the five zones of the channel system. The task of this appendix is to substitute these expressions into eqn (66) and (67) and obtain closed form expressions for critical mobility in terms of known system parameters.

9.3.1 Non-propagating CP

For this case the channel system experiences three zones at equilibrium. The microchannels would have the same BGE concentration as in the supplying reservoir and the nanochannel would have a lower BGE concentration given by eqn (24). The only interfaces at which focusing is possible will be the two microchannel–nanochannel interfaces. Substituting eqn (24) into eqn (66) and noting \( c_0^{\text{d},+} h_0^* \gg 1 \) (microchannel has low Dukhin number) leads to the following expressions for the critical mobility at different channel zones:

\[
v_i^{\text{crit}} = \left[ 1 + \left( \frac{v_i^2}{c_0^{\text{d},+} h_0^*} \right)^{-1} \right]^{\frac{1}{2}}, \quad v_i^{\text{crit}} = 1
\]

Note that for non-propagating CP there are no enrichment or depletion shocks, therefore, we do not distinguish between anode and cathode side reservoir regions when discussing possible focusing.

9.3.2 Propagating CP, \( v_i^2 \geq 1 \)

For this case the BGE concentrations are given in eqn (40) and (50). For the microchannel–nanochannel interfaces and the enrichment shock we can substitute these results into eqn (66) which yields:

\[
v_i^{\text{crit}} = \frac{c_0^{\text{d},+} h_0^*}{v_i^2}, \quad v_i^{\text{crit}} = \frac{h_0^*}{v_i^2} 2^{v_i^2} - 1, \quad v_i^{\text{crit}} = 1, \quad v_i^{\text{crit}} = \frac{2v_i^2 - 1}{v_i^2}
\]

For the depletion shock, we can substitute eqn (36) and (40) into eqn (67) to yield:

\[
v_i^{\text{crit}} = 2^{v_i^2} - 1, \quad v_i^{\text{crit}} = \frac{2v_i^2}{c_0^{\text{d},+} h_0^*}
\]

9.3.3 Propagating CP, \( v_i^2 < 1 \)

For this case the BGE concentrations are given in eqn (57). For the microchannel–nanochannel interfaces and enrichment shock, we can substitute these into eqn (66) resulting in:

\[
v_i^{\text{crit}} = c_0^{\text{d},+} h_0^*, \quad v_i^{\text{crit}} = h_0^*, \quad v_i^{\text{crit}} = 1, \quad v_i^{\text{crit}} = v_i^2
\]

For the depletion shock, we can substitute eqn (57) and (58) into eqn (67) to yield:

\[
v_i^{\text{crit}} = v_i^2, \quad v_i^{\text{crit}} = \frac{v_i^2}{c_0^{\text{d},+} h_0^*}
\]
Fig. S1 Worked example of predictions for anionic analyte behavior. Step 1: Compute $v_\text{crit}^{(x)}$ for each region (reservoir, depletion, nanochannel and enrichment if propagating CP, reservoir and nanochannel if non-propagating CP) in the system. Note that $v_\text{crit}^{(x)}$ in the depletion region is lower than $v_\text{i}$ (indicated by $\sim 0$ in the above figure) for most practical analytes and that $v_\text{crit}^{(x)}$ in the nanochannel is unity by definition. $v_\text{crit}^{(x)}$ in the reservoir and enrichment regions depends only on the background electrolyte and channel properties. Step 2: Check for interfaces where concentration can increase by looking for higher $v_\text{crit}^{(x)}$ upstream ($\text{upstr}$) than downstream ($\text{dwnstr}$). If $v_\text{crit}^{(x) \upstr} > v_\text{crit}^{(x) \downstr}$ then anionic analytes will slow as they pass through the interface, resulting in an increase in concentration. Step 3: Check the value of $v_\text{i}$ for the analyte being considered. If $v_\text{crit}^{(x) \upstr} > v_\text{i} > v_\text{crit}^{(x) \downstr}$ then the analyte will stack or focus at the interface in question. As per the discussion in sections 6.1–6.5, anionic analytes can focus at the depletion shock or either microchannel–nanochannel interface. However, anionic analytes can only ever stack at the enrichment shock.

References


5 Y.-C. Wang and J. Han, *Lab Chip*, 2008, **8**, 392–394.


32 J. Han, J. Fu and R. B. Schoch, *Lab Chip*, 2008, **8**, 23–33.
